Beta-Gamma Directional Correlation in La^{140}

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The directional correlation between the nonunique first-forbidden outer beta group of end-point energy 2.175 MeV in La^{140} and the 1.6-MeV cascade gamma ray in Ce^{140} has been measured at nine beta energies. The measured beta-gamma correlation coefficient ϵ ranges from (0.080 ± 0.010) to (0.111 ± 0.020) in the beta-energy interval 1.6 to 2 MeV. Taking account of the existing data on shape correction factor and also the beta-gamma circular polarization coefficient ω for $\theta=160°$ and $W=4.2$, the relevant first-forbidden beta-decay matrix-element ratios have been extracted; the ratios of the matrix elements are (in Kotani's notation) $x=-0.33\pm0.15$, $u=-0.06\pm0.06$, and $Y=-1.50\pm0.20$. Absolute values of the matrix elements were obtained from the observed ft value as $|\int B_{ij}|/R=(3.3\pm0.5)\times10^{-2}$, $|\int \mathbf{r}|/R=(1.3\pm0.6)\times10^{-2}$, $|\int i\boldsymbol{\sigma}\times\mathbf{r}|/R=(0.2\pm0.2)\times10^{-2}$, $|\int i\boldsymbol{\alpha}|=(4.3\pm1.5)\times10^{-3}$. A cancellation among the vector-type matrix elements is apparently the cause of the deviation from the ξ approximation in this case.

1. INTRODUCTION

THE fundamental characteristics of the β interaction is now well known and it has become of interest to study the various nuclear matrix elements responsible in beta decay. Of particular interest are some $3^-(\beta)2^+(\gamma)0^+$ decays which strongly deviate from the ξ approximation.[1] Such cases are characterized by nonstatistical shapes of the beta spectra, large ft values, and rather large beta-gamma anisotropies. Following the theory developed by Kotani,[2] it is possible to extract the values of the nuclear matrix elements from the knowledge of (1) the ft value of the beta transition; (2) the beta-spectrum shape-correction factor $C(W)$; (3) the energy dependence of the beta-gamma directional correlation coefficient ϵ, and (4) the energy or the angular dependence of the beta-gamma circular polarization correlation coefficient ω.

Earlier investigations[3–9] of nonunique first-forbidden β transitions in Eu^{152}, Eu^{154}, and Sb^{124} have shown that a large ft value results from a suppression of the nuclear matrix elements of tensor rank $\lambda=1$ (i.e., $\int \mathbf{r}$, $\int i\boldsymbol{\sigma}\times\mathbf{r}$, and $\int i\boldsymbol{\alpha}$) relative to the $\int B_{ij}$ matrix element of tensor rank $\lambda=2$. Such kind of suppression of $\lambda=1$ rank matrix elements is characteristic of selection rule effect, either due to the j forbiddenness in the nuclear shell or due to the K forbiddenness in deformed nuclei.

The outer beta group in La^{140} with an end-point energy of 2.175 MeV is nonunique first forbidden[10] (see Ref. 10, Fig. 1 for the decay scheme of La^{140}). However, its $\log ft=9.5$ somewhat exceeds the typical value for most nonunique first-forbidden beta transitions obeying the ξ approximation.

The spin of the ground state of La^{140} has been measured[11] to be $J=3$ and the first excited state of the even-even nucleus Ce^{140} at 1.6 MeV is known to be of character 2+. Thus, the 2.175-MeV $\beta\rightarrow$ 1.6-MeV γ cascade in the decay of La^{140} has a sequence $3^-(\beta)2^+(\gamma)0^+$ and is, therefore, similar to the cases mentioned above. It is of interest to trace the causes of the large $\log ft$ value which is an indication that this beta transition deviates from the ξ approximation. The present investigation aims at determining the beta-gamma directional correlation in La^{140} in the 2.175-MeV beta group and the 1.6-MeV cascade γ ray. The beta spectrum of this decay has a nonstatistical shape[1] given by the shape correction factor $C(W)=q^2+0.845p^2+10\pm5$. The beta-gamma circular polarization correlation coefficient ω has been recently measured by Estulin and Petushkov[12] for $\theta=160°$ and $W=4.2$ to be $\omega=0.15\pm0.09$. Combining these data with the present measurements on the energy dependence of ϵ, a plausible set of relevant nuclear matrix elements responsible for this beta decay has been extracted.

2. EXPERIMENTAL

The source used for the present work was produced by irradiating spectroscopically pure metallic La^{139} with thermal neutrons in the APSARA Reactor at Trombay. Because of the short half-life (40 h) several irradiations were necessary. The source was prepared by depositing a drop of lanthanum chloride solution on a 0.9 mg/cm² Mylar foil over a diameter of about 3 mm.

The vacuum chamber and the detector arrangement have been described in an earlier publication.[7] A 3-in.-diam×3-in.-thick NaI(Tl) crystal coupled to a DuMont 6363 photomultiplier was used for detecting the 1.6-MeV gamma ray.

The integral β–γ correlation was first measured. These measurements were carried out at beta energy

[1] L. M. Langer and D. R. Smith, Phys. Rev. **119**, 1308 (1960).
[2] T. Kotani, Phys. Rev. **114**, 795 (1959).
[3] S. K. Bhattacherjee and S. K. Mitra, Nuovo Cimento **16**, 175 (1960).
[4] J. W. Sunier, P. Debrunner, and P. Scherrer, Nucl. Phys. **19**, 62 (1960).
[5] H. J. Fischbeck and R. G. Wilkinson, Phys. Rev. **120**, 1762 (1960).
[6] H. Dulaney, Jr., C. H. Braden, and L. D. Wyly, Phys. Rev. **117**, 1092 (1960).
[7] S. K. Bhattacherjee and S. K. Mitra, Phys. Rev. **126**, 1154 (1962).
[8] R. M. Steffen, Phys. Rev. **124**, 145 (1961).
[9] P. Alexander and R. M. Steffen, Phys. Rev. **124**, 150 (1961).
[10] W. M. Currie, Nucl. Phys. **32**, 574 (1962).
[11] F. R. Peterson and N. A. Shugart, Bull. Am. Phys. Soc. **5**, 343 (1960).
[12] I. V. Estulin and A. A. Petushkov, Nucl. Phys. **36**, 334 (1962).

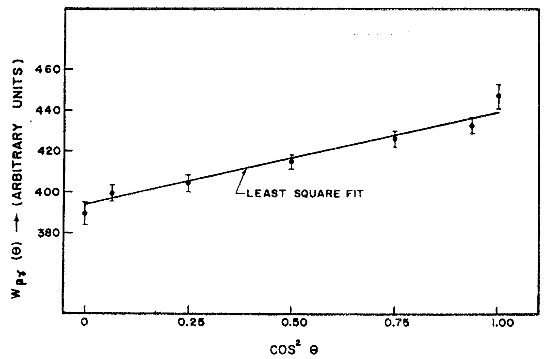

Fig. 1. Integral beta-gamma directional correlation in La[140] with beta energy above 1.65 MeV. In the above plot the beta-gamma coincidences have been plotted as a function of $\cos^2\theta$. The observed correlation coefficient is $\epsilon=+(0.072\pm0.008)$ which when corrected for geometry gives $\epsilon=+(0.081\pm0.009)$.

above 1.65 MeV. The differential anisotropy was measured in the beta-energy region 1.6 to 2 MeV in steps of 50 keV. The data were corrected for geometry and resolution as described before.[7] In La[140] there is an inner beta group with an end-point energy of 1.69 MeV. This possibly introduces some uncertainty of the experimental points at $W=4.15$ and $W=4.23$ (see Fig. 2).

3. RESULTS

The integral beta-gamma correlation function with beta energy above 1.65 MeV after the usual geometrical correction can be expressed as

$$W_{\beta\gamma}(\theta)=1+(0.081\pm0.009)P_2(\cos\theta)+(0.008\pm0.010)P_4(\cos\theta).$$

The small value of the coefficient in the $P_4(\cos\theta)$ term indicates that the parity of La[140] is odd.[2] The results are shown in Fig. 1, where the integral beta-gamma coincidences $W_{\beta\gamma}(\theta)$ are plotted against $\cos^2\theta$.

The experimental differential correlation coefficient for the 2.175-MeV $\beta\rightarrow$ 1.6-MeV γ cascade in La[140]

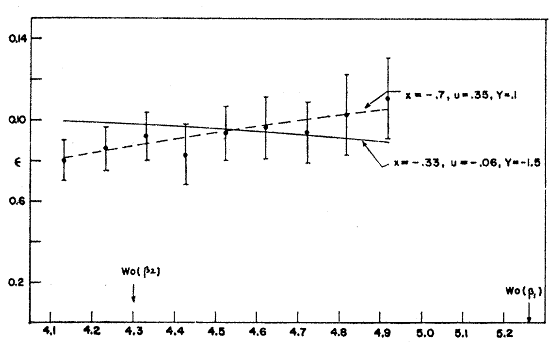

FIG. 2. The beta-gamma directional correlation coefficient ϵ plotted as a function of beta-particle energy W (in units of m_0c^2). The solid curve is a theoretical one with $x=-0.33$, $u=-0.06$, $Y=-1.5$. The dashed curve corresponds to the set $x=-0.7$, $u=0.35$, $Y=0.1$.

measured as a function of the beta energy W (in units of m_0c^2) is shown in Fig. 2. The coefficient ϵ ranges from $+0.08\pm0.01$ to $+0.11\pm0.02$ in the beta-energy region 1.6 to 2 MeV. Our values agree reasonably well with the values reported recently by Steffen.[13] In Fig. 3, the values of the reduced beta-gamma correlation coefficient $\epsilon(p^2/W)^{-1}$ are plotted as a function of the beta-particle energy W. The $\epsilon(p^2/W)^{-1}$ values appear to be independent of energy in the region investigated. The Y values calculated from values of $\epsilon(p^2/W)^{-1}$ using Eq. (4) of Ref. 7 and assuming $x=u=0$ are found to vary from -1.1 at 1.6 MeV to -1.8 at 2 MeV. It is obvious, therefore, that the "modified B_{ij} approximation" does not hold good in this decay of La[140] and it is necessary to take into account the contribution of x and u. To extract the values of the nuclear matrix elements the data were analyzed by an electronic computer. The range of values tried for x and u was from -1 to $+1$ in steps of 0.05 and the range of Y was from -2 to $+2$ in steps of 0.05. The error

TABLE I. Sets of matrix element parameters x, u, and Y obtained by computer which also fits ω or $C(W)$.

x	u	Y	$\epsilon(W)$[a]	$C(W)$[a]	ω[a]
−0.85	−0.30	−1.85	+	+	−
−0.65	−0.15	−1.55	+	+	−
−0.55	−0.05	−1.30	+	0	−
−0.30	0.10	−0.95	+	0	+
−0.25	0.15	−0.80	+	0	++
−0.20	0.20	−0.65	+	−	++
−0.20	0.30	−0.30	++	−	+
−0.15	0.40	−0.05	++	−	0
−0.90	0.25	0	++	+	−
−0.70	0.35	0.10	++	+	−
−0.50	0.50	0.40	++	++	−
−0.70	0.60	0.80	++	+	−

[a] ++ indicates very good fit; + indicates satisfactory fit; 0 indicates poor fit; − indicates no fit.

limits given to the computer were much larger than the actual ones. A very large number of sets were printed showing that the β–γ directional correlation data can be fitted by a wide variety of choices for x, u, and Y. Some of them which also fit the experimental shape factor or the beta-gamma circular polarization correlation coefficient ω were presented in Table I. It is seen that even though there are many sets which fit the directional correlation data, it is difficult to find a unique set which would fit all the three experimental quantities rather satisfactorily. The set $x=-0.25$, $u=0.10$, $Y=-0.95$ seems to give a somewhat approximate fit to all the observed quantities. A set $x=-0.7$, $u=0.35$, $Y=0.1$ gives excellent fit to the directional correlation data as shown in Fig. 2; it also fits the shape correction factor but it does not at all agree with the measured β–γ circular polarization correlation[12] data. The set of values obtained by the computer only gives an indication that Y should be between -0.90

[13] R. M. Steffen, Bull. Am. Phys. Soc. 4, 335 (1961).

and -1.80, to have agreement with all the observed quantities. To determine the matrix elements more precisely, use was made of a graphical method as described below. The expressions for $\epsilon(p^2/W)^{-1}$, ω, and $C(W)$ are quadratic forms in x, u and Y.[7,9] For a particular value of Y, these expressions are reduced to equations quadratic only in x and u, and curves corresponding to them can be plotted in the x–u plane. A common point or region of intersection, if any, would be the desired solution. While applying this procedure, we used the theoretical equations[7,9] expressing the following experimentally determined quantities: (1) $\epsilon(p^2/W)^{-1}$ at 1.65 MeV; (2) $\epsilon(p^2/W)^{-1}$ at 1.95 MeV; (3) $\omega=0.15\pm0.09$ as reported by Estulin and Petushkov,[12] and (4) ratio of $C(W)$ at 1.6 and 2 MeV evaluated from Langer's shape correction factor measurement.[1]

The equations thus obtained were graphically plotted for each value of Y in the region -0.9 to -1.8 in steps of 0.1. For $Y=-1.5$ the best agreement was found. This is partly shown in Fig. 4 where the curves for ω and $C(W)$ are shown by vertical and horizontal shaded lines, respectively. The curve representing ϵ, shaded slantingly in Fig. 4, corresponds to the beta energy 1.8 MeV. This has been done to show the fitting at an intermediate energy. These curves have a common region of intersection around $x=-0.33$ and $u=-0.06$. The theoretical values of ϵ and $\epsilon(p^2/W)^{-1}$ for the set $x=-0.33$, $u=-0.06$, $Y=-1.5$, are shown in Figs. 2 and 3. The fit with the experimentally observed ϵ's is not really very satisfactory but this is the best which could be obtained while trying to fit the shape correction factor[1] and the β–γ circular polarization correlation coefficient[12] also. A precision measurement of ω, particularly its angular dependence, would help in getting a unique set. Estulin and Petushkov[12] tried to fit their β–γ circular polarization data with an integral β–γ directional correlation measurement[14] at an average beta energy and obtained two sets of nuclear parameters (i) $x=+0.7$, $u=+1.4$, $Y=+2.3$ and (ii) $x=-0.35$, $u=-0.20$, $Y=-1.64$. Figure 3 shows that these sets do not agree with our results of $\epsilon(p^2/W)^{-1}$ except probably at the point at which Estulin and Petushkov tried to fit. The agreement of these sets with the experimentally observed shape factor is also not very satisfactory. The shape correction factor calculated for the set $x=-0.33$, $u=-0.06$, $Y=-1.5$ are compared with the experimentally observed shape factor of Langer and Smith in Fig. 5. They have been normalized with the mean experimental value of $C(W)$ at $W=4.52$.

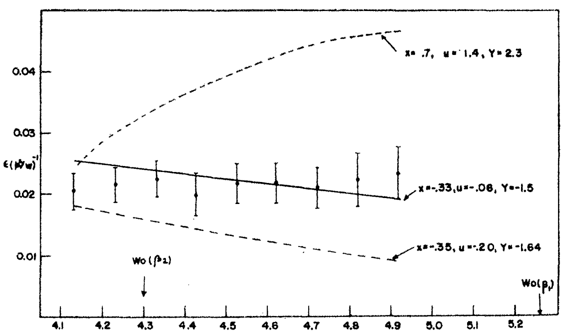

FIG. 3. The "reduced" correlation coefficient $\epsilon(p^2/W)^{-1}$ plotted as a function of beta energy W. The solid curve is Kotani's theoretical expression with $x=-0.33$, $u=-0.06$, and $Y=-1.5$. The two dashed curves correspond to the sets (1) $x=0.7$, $u=1.4$, $Y=2.3$ and (2) $x=-0.35$, $u=-0.20$, $Y=-1.64$ given by Estulin and Petushkov (Ref. 12).

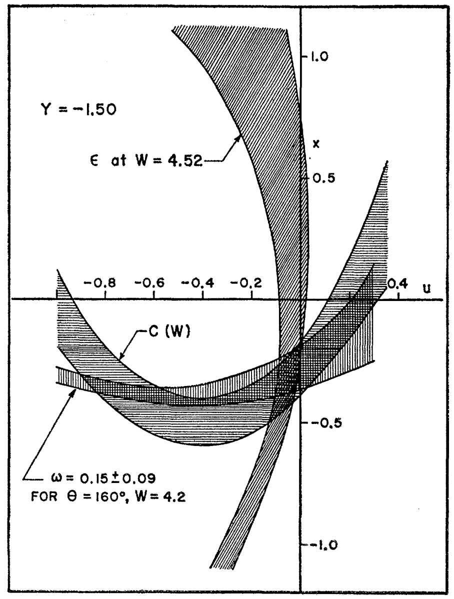

FIG. 4. Plot of x versus u for $Y=-1.5$. The three sets of curves correspond to (1) ϵ at $W=4.52$, (2) $\omega=0.15\pm0.09$ measured by Estulin and Petushkov (Ref. 12) for $\theta=160°$ and $W=4.2$, (3) ratio of $C(W)$ at 1.6 and 2 MeV obtained from Langer's shape-factor measurement (Ref. 1). The curves are shaded in different ways. The common region of intersection is around $x=-0.33$, $u=-0.06$.

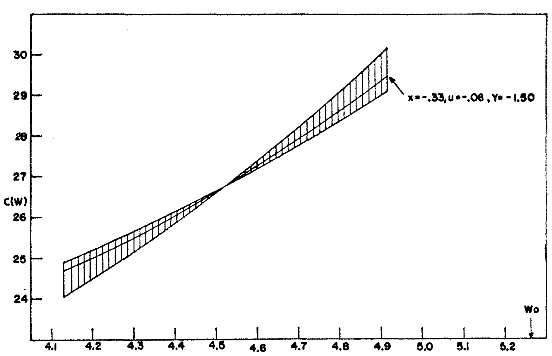

FIG. 5. Comparison of the theoretical shape correction factor $C(W)$ for the set $x=-0.33$, $u=-0.06$, $Y=-1.5$ and the measured shape correction factor of Langer and Smith (Ref. 1). Values of $C(W)$ have been normalized with the mean experimental value at $W=4.52\ m_0c^2$. The shaded area denotes the error assigned to the experimental shape measurement.

[14] V. P. Rudakov, Izv. Akad. Nauk S.S.S.R., Ser. Fiz. 24, 1124 (1960).

The agreement with the measured shape factor is excellent.

From the above analysis, a plausible set for the values of x, u, and Y responsible for this beta decay may be summarized as $x=-0.33\pm0.15$, $u=-0.06\pm0.06$, $Y=-1.5\pm0.2$. The $\log ft$ value[1] of the 2.175-MeV beta transition in La^{140} is 9.5. The correction for the nonstatistical shape of the beta spectrum gives the value $f_ct=10^{9.87}$ sec or in natural units ($\hbar=m_0=c=1$) $f_ct=5.74\times10^{30}$. This determines the standard matrix element $\int B_{ij}=5.38\times10^{-4}$. We have used $C_A=-1.21C_V$, $C_V=g=2.97\times10^{-12}$ (in natural units). The nuclear radius, $R=1.21\times10^{-13}A^{1/3}$ cm, in natural units becomes $R=1.62\times10^{-2}$ for La^{140}. Hence, the absolute values of the nuclear matrix element ratios can be computed in a form which is independent of any system of units and is given as follows:

$$\left|\int B_{ij}\right|\Big/R=(3.3\pm0.5)\times10^{-2},$$

$$\left|\int \mathbf{r}\right|\Big/R=(1.3\pm0.6)\times10^{-2},$$

$$\left|\int i\boldsymbol{\sigma}\times\mathbf{r}\right|\Big/R=(0.2\pm0.2)\times10^{-2},$$

$$\left|\int i\boldsymbol{\alpha}\right|=(4.3\pm1.5)\times10^{-3}.$$

4. DISCUSSION

The ratios of the matrix elements in the 2.175-MeV nonunique first-forbidden beta decay in La^{140} clearly indicate that all the matrix elements are more or less of equal magnitude. For perfect overlap of the initial and final nuclear wave functions, the values of

$$\left|\int B_{ij}\right|\Big/R,\quad \left|\int \mathbf{r}\right|\Big/R,\quad \text{and}\quad \left|\int i\boldsymbol{\sigma}\times\mathbf{r}\right|\Big/R$$

would be of the order unity, whereas $|\int i\boldsymbol{\alpha}|$ would be about 0.1. In comparison to a "normal" nonunique first-forbidden beta transition having $\log ft\approx7$, the $\lambda=1$ type matrix elements are reduced by a factor of about 10; the $\lambda=2$ type matrix element, i.e., $\int B_{ij}$, is also reduced approximately by the same factor in comparison to a "normal" unique first-forbidden beta transition. So in this case we do not have an inhibition of the $\lambda=1$ type matrix elements in preference to the $\lambda=2$ type, which is the characteristic feature of a "selection-rule" effect (e.g., Eu^{152}, Eu^{154}, and Sb^{124}). Thus, here in La^{140} we are most likely encountering a case where there is a "cancellation effect" among $\lambda=1$ type matrix elements. However, the cancellation effect alone cannot account for the large $\log ft$ value; the individual $\lambda=1$ type matrix elements are themselves relatively small compared to normal nonunique transitions. Further, there cannot be a j-selection rule effect in La^{140} since the 57th proton and the 83rd neutron occupy different major shells.

The measured spin of $I=\frac{7}{2}$ in La^{139} indicates that the 57th proton occupies a $g_{7/2}$ state; also the measured spin of $I=\frac{7}{2}$ in Ce^{141} indicates that the 83rd neutron is in the $f_{7/2}$ state. In a pure shell model the ratio $\int i\boldsymbol{\sigma}\times\mathbf{r}/\int\mathbf{r}$ is predicted theoretically[15,16] which does not involve the radial wave functions but is given only by angular momentum couplings. For the La^{140} configuration of $(1g_{7/2}, 2f_{7/2})$ the predicted value of $\int i\boldsymbol{\sigma}\times\mathbf{r}/\int\mathbf{r}=-8$ in contrast to the experimental value of the ratio lying within limits $0\leqslant\int i\boldsymbol{\sigma}\times\mathbf{r}/\int\mathbf{r}\leqslant0.53$. Thus, the extreme single-particle shell model does not predict the correct ratio of the quantity $\int i\boldsymbol{\sigma}\times\mathbf{r}/\int\mathbf{r}$.

Further, if one assumes the validity of the conserved vector current hypothesis, a relationship can be established connecting the relativistic matrix element $\int i\boldsymbol{\alpha}$ with the nonrelativistic matrix element $\int\mathbf{r}$ as has been recently given by Fujita[16]:

$$\int i\boldsymbol{\alpha}\Big/\int\mathbf{r}=\Lambda_{\text{cvc}}\xi,$$

where $\Lambda_{\text{cvc}}=2.4+(W_0-2.5)(A^{1/3}/Z)$ for e^- emission. The predicted value of Λ_{cvc} for the 2.175-MeV beta decay of La^{140} is 2.65 ($\xi_{\text{La}}=13$), whereas the experimental value of this ratio lies within the limits $0.7\leqslant\Lambda_{\text{exp}}\leqslant4.0$. Thus, within the limits of experimental error the prediction of the conserved vector current hypothesis holds good for this beta decay. However, in the conventional bare nucleon coupling theory, Ahrens and Feenberg[17] predict

$$\Lambda_{\text{AF}}=1+(W_0-2.5)(A^{1/3}/Z).$$

For La^{140} decay, $\Lambda_{\text{AF}}=1.25$. So the limits of the errors in the experimental value of Λ_{exp} is such that it does not distinguish between the predictions of the two theories.

It should be pointed out that the most plausible set of the matrix-element ratios given here was determined by using the results of the beta-gamma circular polarization correlation measurement which was done only at one angle and at an average beta energy over the spectrum.[12] For the combination of values of x, u, and Y given for this decay in this work, the dependence of ω on the angle between the beta and gamma rays is very steep for all beta energies. So a detailed measurement on the angular or energy dependence of ω is very desirable, which will enable one to evaluate the matrix elements with better accuracy. This is important to enable one to distinguish between the predictions of the conserved vector current hypothesis and that due to the conventional theory. Further, an accurate

[15] G. E. Lee-Whiting, Phys. Rev. **97**, 463 (1955).
[16] J. Fujita, Phys. Rev. **126**, 202 (1962).
[17] T. Ahrens and E. Feenberg, Phys. Rev. **86**, 64 (1952).

measurement of the longitudinal polarization of the beta particles will establish the cancellation effect in this decay because, in such a case, the longitudinal polarization will be substantially different from the v/c law.

Note added in the proof. After this work was sent for publication, a similar work has been reported by Fischbeck and Newsome in Bull. Am. Phys. Soc. **8**, 332 (1963). Their measurements are in very good agreement with our results.

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Octupole Deformation in Even-Even Medium Mass Nuclei

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The stability of octupole deformation for even-even medium mass nuclei with small spheroidal deformation has been studied in order to explain the existence of odd-parity excited states in some of them. The total single-particle energy is calculated by an exact diagonalization of the Nilsson Hamiltonian with octupole deformation, neglecting the residual two-particle interaction. Within this framework, it is found that these nuclei are stable against octupole deformation.

In the past years much attention has been given to the experimental investigation of excited states of even-even medium mass nuclei. A systematic occurrence of low-lying odd-parity excited states with spin 3- or 5- in a number of even-even nuclei, with mass number in the range $60 \leqslant A \leqslant 150$, has been observed recently by various workers.[1–5] Most of these odd-parity excited states lie within the energy range of 2 to 3 MeV.

It is known that the nuclei in the region under consideration have spectra of a vibrational kind. These spectra have been interpreted in various ways,[6–11] but the most widely held view is that outside the rotational regions and excluding the few closed-shell nuclei, there are nuclei which have a tendency to deform, but the deformation has fluctuations large compared with the magnitude of the deformation and since these fluctuations in shape have dynamical properties there should be nuclear excitation analogous to waves on the nuclear surface. The known spectra have been interpreted in terms of quadrupole surface vibrations. Since the expressions for the reduced transition rate in the vibration and rotation models are identical, one can assign a mean deformation to these nuclei.[12]

The occurrence of the odd-parity excited state in these nuclei has given rise to the question of the existence of octupole deformation in them. The purpose of the present work is to see whether such a deformation is energetically favored for some nuclei in this range. The stability of octupole deformation in some nuclei lying in the rare-earth and actinide region and possessing a particular value of spheroidal deformation have been studied earlier by Dutt and Mukherjee.[13] But the question of the stability of octupole deformation for nuclei lying in the region under consideration and having different spheroidal deformation has not received attention.

We have calculated the total single-particle energy by an exact diagonalization of Nilsson[14] Hamiltonian with an additional term for the octupole deformation. The Hamiltonian we have used may, therefore, be written as

$$H=\chi\hbar\omega_0{}^0\left\{\frac{2K}{a_2}(\tfrac{1}{2}\nabla^2-\tfrac{1}{2}r^2)-Kr^2\left[9\left(\frac{3}{140\pi}\right)^{1/2}a_3Y_1{}^0(\theta,\phi)+Y_2{}^0(\theta,\phi)+\frac{a_3}{a_2}Y_3{}^0(\theta,\phi)\right]-2\mathbf{L}\cdot\mathbf{s}-\mu\mathbf{L}^2\right\}, \quad (1)$$

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[1] O. Hansen and O. Nathan, in *Proceedings of Rutherford Jubilee International Conference*, edited by J. E. Birks (Heywood and Company, Ltd., Manchester, 1961), p. 267.

[2] M. K. Ramaswamy, W. L. Skell, D. L. Hutchins, and P. S. Jastram, Phys. Rev. **121**, 553 (1961).

[3] M. Strominger, J. M. Hollander, and G. T. Seaborg, Rev. Mod. Phys. **30**, 585 (1958).

[4] R. L. Robinson, E. E. Eichler, and Noah. R. Johnson, Phys. Rev. **122**, 1863 (1961).

[5] M. N. Rao, Nucl. Phys. **33**, 182 (1962).

[6] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **27**, No. 16 (1953).

[7] G. Scharff-Goldhaber and J. Weneser, Phys. Rev. **98**, 212 (1955).

[8] A. S. Davydov and G. F. Fillipov, Nucl. Phys. **8**, 237 (1958).

[9] C. Marty, Nucl. Phys. **1**, 85 (1956); **3**, 193 (1957).

[10] L. Wilets and M. Jean, Phys. Rev. **102**, 788 (1956).

[11] A. S. Davydov and A. A. Chaban, Nucl. Phys. **20**, 499 (1960).

[12] D. M. Brink, Progr. Nucl. Phys. **8**, 97 (1960).

[13] I. Dutt and P. Mukherjee, Phys. Rev. **124**, 888 (1961). One of the authors (SNM) is grateful to Dr. Dutt for useful discussions.

[14] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **29**, No. 16 (1955).